Circuits Test: Show your work and give concise answers.

<table>
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<tr>
<th>Analog (50)</th>
<th>Communications (50)</th>
<th>Total Points</th>
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</thead>
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**Analog:** In this problem, analyze the small-signal behavior of some common transistor configurations and show that you understand how to use these circuits. Express all answers in pole-zero form in the simplest form to receive full credit.

1. Determine the unity current gain frequency for the first-order NMOS model in Fig. 1. Assume the source is grounded, a voltage source drives the gate, and the output is the drain. (5)

2. For the source follower stage in Fig. 2, derive the input impedance based on the first-order NMOS model. (5)

3. For the source follower stage in Fig. 2, derive the output impedance based model in Fig. 1. (10)

4. For the source follower stage, derive the voltage transfer function. (10)

5. What is the 3dB bandwidth of the source follower stage? (5)

6. For the common source stage, derive the voltage transfer function. (10)

7. If the first order transistor model included a gate-drain capacitance, Cgd, where would the zero be located in the common source voltage transfer function? (5)
Problem 2: RF and Communication Circuits

Impedance Matching: The following questions refer to Fig. 1.

1) If \( Z_L = 40 + j30 \Omega \) and \( Z_s = 50 \Omega \), which impedance matching network would you use? Justify your answer. (5)

2) Find the impedance, \( X \), and susceptance, \( B \), for the matching network. (15)

3) Assume the matching network is not used and find the return loss as seen from the source in dB. (5)

Communication Circuits: The receiver chain is required for a new radio. It consists of an input attenuator, an RF amplifier, a SSB mixer, a bandpass IF filter, and a bandpass ADC.

4) Your company is designing a receiver chain and has specified the building blocks in Fig. 2. What is the receive chain gain in dB? (5)

5) What is the single sideband noise figure of the receive chain seen at the antenna? (10)

6) Now consider the RF amplifier in Fig. 3 that is used in your receiver chain. What is the IIP3 of the BJT amplifier? Assume \( V_r = 25 \text{mV} \). Remember that the Taylor series

\[
f(x) = f(a) + f'(x-a) + f''(x-a)/2 + f'''(x-a)/6 \quad \text{and} \quad \text{IIP3} = \sqrt[3]{\frac{4}{3} \frac{g_1}{g_3}}
\]
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**Analog:** in this problem, analyze the small-signal behavior of some common transistor configurations and show that you understand how to use these circuits. Express all answers in pole-zero form in the simplest form to receive full credit.

![Circuits Diagram](image)

**Fig. 1. First Order NMOS Model**

**Fig. 2. Source Follower Stage**

**Fig. 3. Common Source Stage**

1.) Determine the unity current gain frequency for the first-order NMOS model in Fig. 1. Assume the source is grounded, a voltage source drives the gate, and the output is the drain. (5)

\[ f_T = \frac{1}{2\pi C_{gs}} \]

2.) For the source follower stage in Fig. 2, derive the input impedance based on the first-order NMOS model. (5)

\[ Z_{in} = R_s + R_f + \frac{1}{sC_{gs}} \left( 1 + g_m R_f \right) \]

3.) For the source follower stage in Fig. 2, derive the output impedance based model in Fig. 1. (10)

\[ Z_{out} = \frac{R_f}{1 + g_m R_f} \left[ \frac{1 + sC_{gs} R_f}{1 + sC_{gs} \left( R_s + R_f \right)} \right] \]

4.) For the source follower stage, derive the voltage transfer function. (10)

\[ A_v = \frac{g_m R_f}{1 + g_m R_f} \left[ \frac{1 + sC_{gs} R_f}{1 + s C_{gs} R_s} \right] \]

5.) What is the 3dB bandwidth of the source follower stage? (5)

\[ f_{3dB} = \frac{1}{2\pi} \frac{1 + g_m R_f}{R_f C_{gs}} \]

6.) For the common source stage, derive the voltage transfer function. (10)
\[ A_v = g_m R_d \frac{1}{1 + sC_{gs} R_t} \frac{1}{1 + sC_{L} R_d} \]

7. If the first order transistor model included a gate-drain capacitance, Cgd, where would the zero be located in the common source voltage transfer function? (5)

\[ f_z = \frac{1}{2\pi} \frac{g_m}{C_{gd}} \]

**Problem 2: RF and Communication Circuits**

Impedance Matching: The following questions refer to Fig. 1.

![Fig. 1. Impedance Matching Networks](image)

1. If \( Z_L = 40 + j30 \Omega \) and \( Z_s = 50 \Omega \), which impedance matching network would you use? Justify your answer. (5)

The real part of the load impedance is less than the source impedance. Therefore, the network on the left should be used.

2. Find the impedance, \( X \), and susceptance, \( B \), for the matching network. (15)

\[ Q = \sqrt{\frac{50}{40} - 1} = \sqrt{0.25} = 0.5, \quad X + 30 = |QR| = 0.5 \cdot 40 = \pm 20 \rightarrow X = -10, -50, \]

\[ B = \frac{1}{\Im \{Z_L\} + X + \left(\Re \{Z_L\}\right)^2} - \frac{1}{\Im \{Z_L\} + X + \left(\Re \{Z_L\}\right)^2} - \frac{1}{100} \cdot \frac{1}{100} - \frac{\Re \{Z_L\}}{20} - \frac{\Re \{Z_L\}}{20} \]

3. Assume the matching network is not used and find the return loss as seen from the source in dB. (5)

\[ \Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{-10 + j30}{90 + j30} = \frac{-900 + j2700 + j300 + 900}{8100 + 900} = \frac{j}{3} \]
\[ RL = 20 \log_{10} |\Gamma| = -20 \log_{10} (3) = -9.5 \text{dB} \]

**Communication Circuits:** The receiver chain is required for a new radio. It consists of an input attenuator, an RF amplifier, a SSB mixer, a bandpass IF filter, and a bandpass ADC.

![Receiver Chain Diagram]

**Fig. 2. Receiver Chain**

<table>
<thead>
<tr>
<th>G1 = 20dB</th>
<th>G2 = 0dB</th>
<th>G3 = -6dB</th>
<th>NF1 = 8dB</th>
<th>NF2 = 10dB</th>
<th>NF3 = 6dB</th>
<th>NF4 = 30dB</th>
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![RF Amplifier Diagram]

**Fig. 3. RF Amplifier**

4) Your company is designing a receiver chain and has specified the building blocks in Fig. 2. What is the receive chain gain in dB? (5)

\[ G = G_1 G_2 G_3 = 10 \cdot 0.5 = 5 \rightarrow 14 \text{dB} \]

5) What is the single sideband noise figure of the receive chain seen at the antenna? (10)

\[ F = 2 + \frac{F_1 - 1}{G_1} + \frac{F_2 - 1}{G_1 G_2} = 2 + 3 + \frac{10}{10} + \frac{4}{10} = 6.4 \rightarrow 8 \text{dB} \]

6) Now consider the RF amplifier in Fig. 3 that is used in your receiver chain. What is the IIP3 of the BJT amplifier? Assume \( V_T = 25 \text{mV} \). Remember that the Taylor series is

\[ f(x) = f(a) + f'(x-a) + f''(x-a)/2 + f'''(x-a)/6 \text{ and } IIP3 = \left[ \frac{4}{3} \right] g_3 \]

\[ V_o = V_{cc} - I_c R_i = V_{cc} - I_c e^{u/R} R_i \]

\[ IIP3 = \sqrt[3]{\frac{4}{3} 6v_T^2} = 2\sqrt[3]{2} v_T \approx 70 \text{mV} \]