1. (a) For graphene we have, as stated in the problem, \( E = E_0 + \frac{\sqrt{3}}{2} \hbar \omega \left( \frac{k'}{2} \right) \) for the conduction band. Let \( k' = \alpha k \) and \( \alpha \equiv \frac{\sqrt{3}}{2} \hbar \omega \Rightarrow E = E_0 + \alpha k' \). Then states with energy between \( E_0 \) and \( E \) i.e., in the conduction band for \( E > E_0 \), correspond to those with \( 0 < k' < \frac{E-E_0}{\alpha} \). The area in \( k \) space occupied by these states is \( k \sqrt{\frac{E-E_0}{\alpha}} \). Then the total number of such electronic states is, for an \( L \times L \) sheet of graphene, given by

\[
N = 2 \frac{\pi k^2}{L_{\parallel}^2} \frac{E-E_0}{\alpha} \times \frac{E-E_0}{\alpha} = \frac{12 \pi}{\alpha L_{\parallel}^2} \left( \frac{E-E_0}{\alpha} \right)^2 \frac{L_{\parallel}^2}{\pi} \frac{E-E_0}{\alpha} \]

The density of states is then given by \( \sigma(E) = \frac{1}{E_0} \frac{dN}{dE} \)

\[
\Rightarrow \sigma(E) = \frac{6}{\pi \omega^2} (E-E_0), \quad \text{or} \quad \sigma(E) = \frac{3L^2}{\pi \hbar^2 \alpha^2} (E-E_0) \quad \text{for} \quad E > E_0
\]

(b) Confinement to a region of width \( d \) in the \( x \)-direction with infinite potential energy barriers yields \( k_x = \frac{\pi n}{d} \), so that the energy in the graphene conduction band is given by

\[
E = E_0 + \frac{\sqrt{3}}{2} \hbar \omega \sqrt{k_x^2 + k_y^2} = E_0 + \frac{\sqrt{3}}{2} \hbar \omega \sqrt{\left( \frac{\pi n}{d} \right)^2 + k_y^2} \quad \text{with} \quad \alpha \equiv \frac{\sqrt{3}}{2} \hbar \omega \text{ as before.}
\]

The lowest allowed energy for a given \( n \) is then

\[
E_n = E_0 + \frac{\sqrt{3}}{2} \hbar \omega \left( \frac{\pi n}{d} \right)
\]

For the confined levels, the states with energy between \( E_n \) and \( E_{n+1} \) are

\[
E_g = \sqrt{3} \hbar \omega \left( \frac{\pi}{d} \right)
\]

For the graphene valence band (-51 pm in expression for \( E(B) \)), we similarly have \( E_n = E_0 - \frac{\sqrt{3}}{2} \hbar \omega \left( \frac{\pi n}{d} \right) \). The band gap \( E_g \) is then the energy difference between the lowest conduction band state and highest valence band state, which we easily see to be

\[
E_g = \sqrt{3} \hbar \omega \left( \frac{\pi}{d} \right)
\]
c) For each conductor band confined level, the states with energy between $E_n$ and $E$ satisfy $\sqrt{(\frac{2n\hbar^2}{m}) + \frac{k_f^2}{\hbar^2}} < \frac{1}{\alpha} (E - E_n) = \frac{k_f}{a^2} (E - E_n)^2 - (\frac{2\pi}{a})^2$, or $k_f < [\frac{1}{\alpha} (E - E_n)^2 - (\frac{2\pi}{a})^2]^{1/2}$.

The number of $k_f$ occupied by these states has length $2 \sqrt{\frac{1}{\alpha} (E - E_n)^2 - (\frac{2\pi}{a})^2}$.

Then the total number of electronic states for a system of length $L$ is $N = 2 \times 6 \times 2 \left[ \frac{(E - E_n)^2}{\alpha} - (\frac{2\pi}{a})^2 \right]^{1/2} / (2\pi L) = \frac{24E}{2\pi} \left[ \frac{(E - E_n)^2}{\alpha} - (\frac{2\pi}{a})^2 \right]^{1/2}$.

For each confined level, we have again $\delta(E) = \frac{1}{E} \frac{dN}{dE}$ (for the 1-d system, so $\delta(E) = \left[ \frac{(E - E_n)^2}{\alpha} - (\frac{2\pi}{a})^2 \right]^{1/2} E - E_n \right] / (2\pi L)$, or $\delta(E) = \frac{16}{\pi \hbar^2 a^2} \left[ \frac{(E - E_n)^2}{\alpha} - (\frac{2\pi}{a})^2 \right]^{1/2}$.

For all confined levels, we then have

$$\delta(E) = \sum_{E_n < E} \frac{16}{\pi \hbar^2 a^2} \left[ \frac{(E - E_n)^2}{\alpha} - (\frac{2\pi}{a})^2 \right]^{1/2}$$  

with $E_n = E_0 + \frac{\sqrt{2}}{2} \hbar v_l a (\frac{2\pi}{a})$.

n) For this part, we treat the graphene "strip" of part (b) as a one-dimensional ballistic transport channel. We then have, schematically, the following contact/graphene/contact,

```
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Contact</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>$E_n$</td>
</tr>
<tr>
<td>$V/2$</td>
</tr>
</tbody>
</table>
```

In the zero-temperature limit, the conductance becomes nonzero when $E_1$ crosses the Fermi level of the grounded contact (for $V > 0$). This occurs when $\frac{V}{2} = E_1 - E_0$, or $\frac{V}{2} = \frac{\sqrt{2}}{2} \hbar L a (\frac{2\pi}{a})$.

The structure is symmetric, so for $V < 0$, the conductance becomes nonzero for $-V = \sqrt{2} \hbar L a (\frac{2\pi}{a})$.

The value of conductance at this point jumps from zero to the usual quantum of conductance, or $G = \frac{2e^2}{h}$. 