Math Question (equal weight each part) - ECE MS Comp Exam, Fall 2013

Part 1

(i) Solve the following set of coupled differential equations

\[
\frac{dA(z)}{dz} = j\gamma B(z) \quad \quad \quad \frac{dB(z)}{dz} = j\gamma A(z)
\]

subject to the boundary conditions that \(A(0) = C\) and \(B(0) = 0\). (Note that \(j \equiv \sqrt{-1}\))

(ii) Sketch the solutions for \(|A(z)|^2\) and \(|B(z)|^2\) for a value of \(\gamma = \pi/4\).

Part 2

A zero-mean joint Gaussian probability distribution is given by

\[
f_{xy}(x,y) = \frac{1}{2\pi\sigma^2} \exp \left( -\frac{x^2 + y^2}{2\sigma^2} \right).
\]

(i) Derive the probability distribution \(f_z(z)\) for the random variable

\[
z = x^2 + y^2,
\]

which is the squared-magnitude of the joint Gaussian probability distribution and sketch this distribution for \(\sigma^2 = 1\).

(ii) Derive the probability distribution \(f_\phi(\phi)\) for the random variable

\[
\phi = \tan^{-1} \left( \frac{y}{x} \right),
\]

which is the phase of the joint Gaussian distribution and sketch this distribution for \(\sigma^2 = 1\).