230A (FA10) Solution:

(a) The Schrödinger equation is:

\[
\frac{-\hbar^2}{2m} \nabla^2 \phi + V(x) \phi = E \phi
\]

So in region I, \( V(x) = 0 \)

The Schrödinger equation is:

\[
- \frac{\hbar^2}{2m_1} \frac{d^2 \phi}{dx^2} = E \phi
\]

The general solution is:

\[
\phi_1(x) = Ae^{ikx} + Be^{-ikx}, \quad \text{where} \quad \beta = \sqrt{2m_1 E / \hbar}
\]

And in the region II, \( V(x) = V_0 \),

\[
- \frac{\hbar^2}{2m_2} \frac{d^2 \phi}{dx^2} + V_0 \phi = E \phi
\]

The general solution is:

\[
\phi_2(x) = Ce^{\alpha x} + De^{-\alpha x}, \quad \text{where} \quad \alpha = \sqrt{2m_2 (V_0 - E) / \hbar}, \quad V_0 \geq E
\]

(b) Boundary conditions:
1. \( \phi(0) \) continues in region I and II:
   \[ A + B = C + D \]
2. Continuity of probability flux in regions I and II:
   \[
   \frac{1}{m_1} \frac{d \phi_1}{dx} \bigg|_{x=0} = \frac{1}{m_2} \frac{d \phi_2}{dx} \bigg|_{x=0}
   \]
   So \( \frac{i\beta(A - B)}{m_1} = \frac{\alpha(C - D)}{m_2} \)
3. Bloch Theorem:
   \[
   \phi_1(x = a) = \phi_2(x = -b) e^{ikc}, \quad \text{where} \quad c = a + b
   \]
   \[ Ae^{ika} + Be^{-ika} = (Ce^{-\alpha a} + De^{\alpha a}) e^{ikc} \]
4. Continuity of probability flux in \( a \) and \( -b \),
   \[
   \frac{i\beta(Ae^{ika} - Be^{-ika})}{m_1} = \frac{\alpha(Ce^{-\alpha a} - De^{\alpha a})}{m_2} e^{ikc}
   \]

(c,d) Typical solutions of the wavefunction with \( E < V_0 \) and \( E > V_0 \).
230B (FA10) Solution:

(a) The collector current does not change. $I_B$ decreases to half. Therefore, the current gain doubles.

(b) The collector current $I_C$ doubles. The base current does not change. Therefore, the current gain doubles.

(c) The collector current increases by $\exp(q/kT) = 46.8$. The base current increases by the same amount. Therefore, the current gain does not change.

(d) $n_{ieB}$ is strongly dependent on the energy band gap, i.e.,

$$n_{ieB}^2 (SiGe) = n_{ieB}^2 (SiGe) \exp\left(\frac{\Delta E_{g,SeC}}{kT}\right).$$

Therefore, the collector current increases by the factor of 46.8. The base current does not change. The current gain, therefore, increases by the same amount as the collector current.