Math Question

Problem 1

A vector field is given by

\[ \mathbf{V}(x, y, z) = e^x \sin y \mathbf{i} + e^x \cos y \mathbf{j} + 5 \mathbf{k} \]

where \( \mathbf{i}, \mathbf{j} \) and \( \mathbf{k} \) are unit vectors in the \( x, y \) and \( z \) directions respectively. Determine the work done by the vector field on an object moving along a smooth curve from the point \((-\pi, 0, \pi)\) to the point \((\pi, 0, -\pi)\) and then back to the original point. (No credit without justification.)

Problem 2

Find the eigenvectors and eigenvalues of the following matrix

\[ \mathbf{M} = \begin{bmatrix} 3 & 10 \\ 2 & 4 \end{bmatrix} \]

a) Are the eigenvectors orthogonal? (No credit without justification.)
b) Express the vector

\[ \mathbf{a} = \begin{bmatrix} -14 \\ 11 \end{bmatrix} \]

as a linear combination of the eigenvectors and using this decomposition, express the vector matrix product

\[ \mathbf{b} = \mathbf{Ma} \]

as a linear combination of the eigenvectors.