Problem 1
You are using a robot equipped with a camera to localize a chair in your room. The robot is located at position $p_0 = [-1, 1, 0]^T$ with orientation $R_0 = \begin{bmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Assume that the frames of reference of the robot and the camera coincide. Your camera is calibrated and has an intrinsic calibration matrix $K = \begin{bmatrix} 1 & 0 & 100 \\ 0 & 1 & 100 \end{bmatrix}$. You are guessing that your chair is located at $\mu_0 = [1, 1, 0]^T$ and place a Gaussian distribution with identity covariance on your guess. You rotate your robot $30^\circ$ counter-clockwise while translating it by $p_\Delta = \frac{1}{2}[-\sqrt{3}, \sqrt{3} + 1, 0]^T$. You run your chair-detection algorithm on the image received at the new robot pose and detect the chair at pixel location $z = [100, 100]^T$. You know that your algorithm reports detections perturbed by Gaussian noise with zero mean and identity covariance. Use this measurement and the extended Kalman filter to update your prior guess about the chair’s position. **Compute the updated mean $\mu$ and covariance $\Sigma$ of the chair position.**

**Reminders:**
- The pixel coordinates of a point $y \in \mathbb{R}^3$ observed by a camera with position $p \in \mathbb{R}^3$, orientation $R \in \text{SO}(3)$, and intrinsic parameters $K \in \mathbb{R}^{2 \times 3}$ are:
  
  $$z = K\pi(R_{oc}R^T(y - p)) \in \mathbb{R}^2$$
  
  where $R_{oc} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$ and $\pi(x) := \frac{1}{x_3} x \in \mathbb{R}^3$
- A rotation of $\theta$ radians around the $z$-axis can be represented by a rotation matrix:
  
  $$R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
- The a posteriori covariance of the update step of the Kalman filter is
  
  $$\Sigma_{t+1|t+1} = \Sigma_{t+1|t} - \Sigma_{t+1|t}H^T(H\Sigma_{t+1|t}H^T + V)^{-1}H\Sigma_{t+1|t}$$