

## **FACULTY MENTOR**

Yang Zheng

## **PROJECT TITLE**

Convex Relaxations for Global Optimization over Neural Networks

## **PROJECT DESCRIPTION**

Description: Neural networks (NNs) are able to represent complex non-convex functions, and they have shown promising results in many applications, including image recognition, natural language processing, and robotic control. Some recent work has started to incorporate Deep NNs (DNNs) into safety-critical systems, like autonomous driving. Although DNNs obtain unprecedented empirical results, their opacity poses significant challenges, especially in safety-critical systems. It is therefore desirable to provide rigorous analysis and guarantees about the behaviors of DNNs.

In this project, we are interested in solving global optimization problems with the input/output behavior of NNs as constraints. The input/output behavior of NNs is nonlinear and non-convex, making it hard to solve the optimization problem exactly. A variety of convex relaxation techniques, such as interval bound propagation, linear programs, semidefinite programs, and dual relaxation, have been proposed to derive an approximated solution. In this project, the student will investigate different convex relaxation techniques for global optimization problems over NNs. The student will first survey and understand the existing approaches, such as [1]-[6] and the references therein, and then compare their relaxation quality and conservativeness. Finally, based on the survey and comparison, it will be very interesting to propose new relaxation techniques or new ideas on tightening the existing approaches.

This project will be in person.

## **INTERNS NEEDED**

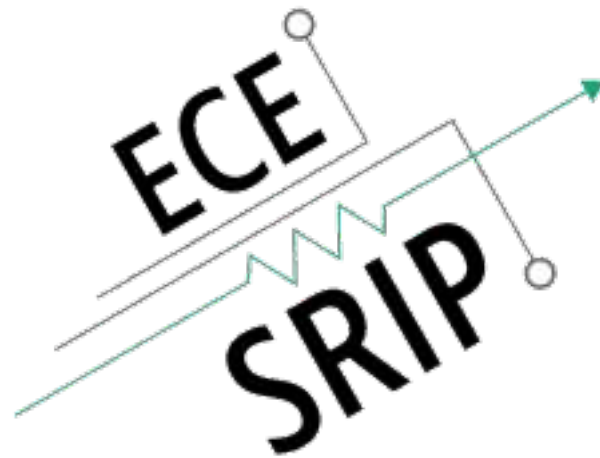
2 Students

## **PREREQUISITES**

1. A strong background in linear algebra and optimization, and good programming skills (e.g., Python, Matlab, Julia) are required

## References

- [1] Strong, C. A., Wu, H., Zeljić, A., Julian, K. D., Katz, G., Barrett, C., & Kochenderfer, M. J. (2020). Global Optimization of Objective Functions Represented by ReLU Networks. arXiv preprint arXiv:2010.03258.
- [2] Salman, H., Yang, G., Zhang, H., Hsieh, C. J., & Zhang, P. (2019). A convex relaxation barrier to tight robustness verification of neural networks. In *Advances in Neural Information Processing Systems* (pp. 9835-9846).
- [3] Singh, G., Ganvir, R., Püschel, M., & Vechev, M. (2019). Beyond the single neuron convex barrier for neural network certification. In *Advances in Neural Information Processing Systems* (pp. 15098-15109).
- [4] Li, L., Qi, X., Xie, T., & Li, B. (2020). SoK: Certified Robustness for Deep Neural Networks. arXiv preprint arXiv:2009.04131.
- [5] Liu, C., Arnon, T., Lazarus, C., Barrett, C., & Kochenderfer, M. J. (2019). Algorithms for verifying deep neural networks. arXiv preprint arXiv:1903.06758.
- [6] Batten, B., Kouvaros, P., Lomuscio, A., & Zheng, Y. (2021). Efficient neural network verification via layer-based semidefinite relaxations and linear cuts. In *30th International Joint Conference on Artificial Intelligence (IJCAI-21)*, accepted.



## **FACULTY MENTOR**

Yang Zheng

## **PROJECT TITLE**

Inner and Outer Approximations for Semidefinite Optimization

## **PROJECT DESCRIPTION**

Description: Convex optimization has profound impacts on many problems in control theory, discrete and nonlinear optimization, theoretical computer science, and machine learning. It is a fundamental tool to ensure efficient, resilient, and safe operations of many engineering systems, such as smart power grid, transportation, robotics, and many others. Optimization in these areas often takes the form of conic optimization, especially semidefinite programs. However, the existing interior-point methods for solving semidefinite programs are not very scalable for large-scale problems. In this project, the student will explore different inner and outer approximation methods to improve the efficiency of solving large-scale semidefinite optimization. One particular focus will be on the balance of approximation quality and numerical efficiency. The student will begin with reading some existing methods, e.g., [1][2][3][4], and then explore new ideas and implement new algorithms.

- [1] Zheng, Y., Sootla, A., & Papachristodoulou, A. (2019). Block factor-width-two matrices and their applications to semidefinite and sum-of-squares optimization. arXiv preprint arXiv:1909.11076.
- [2] Ahmadi, A. A., & Majumdar, A. (2019). DSOS and SDSOS optimization: more tractable alternatives to sum of squares and semidefinite optimization. *SIAM Journal on Applied Algebra and Geometry*, 3(2), 193-230.
- [3] Ahmadi, A. A., Dash, S., & Hall, G. (2017). Optimization over structured subsets of positive semidefinite matrices via column generation. *Discrete Optimization*, 24, 129-151.
- [4] Zheng, Y., Fantuzzi, G., & Papachristodoulou, A. (2021). Chordal and factor-width decompositions for scalable semidefinite and polynomial optimization. *Annual Reviews in Control*.

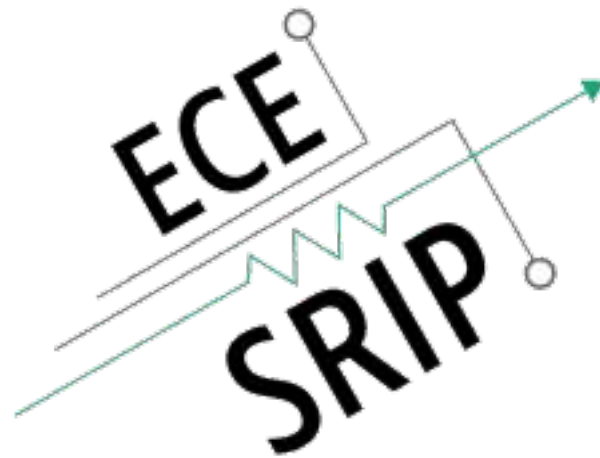
This project will be in person.

## **INTERNS NEEDED**

1 Student

## **PREREQUISITES**

1. A strong background in linear algebra and optimization is required



## **FACULTY MENTOR**

Yang Zheng

## **PROJECT TITLE**

Autonomous Vehicles in Mixed Traffic Systems: Analysis, Control, and Numerical Experiments

## **PROJECT DESCRIPTION**

Description: The emergence of connected and autonomous vehicles (CAVs) promises to revolutionize road transportation systems. In particular, the advancements of CAVs offer new opportunities for traffic control, where autonomous vehicles can receive information from other traffic participants and act as mobile actuators to influence traffic flow internally, thus increasing efficiency and safety on public roads. Both small-scale real-world experiments (e.g., <https://youtu.be/2mBjYZTeaTc>) and large-scale numerical simulations (e.g., <https://flow-project.github.io/>) have shown promising results. Recent work has also started to build a solid theoretical understanding toward of the influence of CAVs in mixed traffic involving both autonomous vehicles and human-driven vehicles.

In this project, the student will begin with reading some existing analysis and control methods of autonomous vehicles in mixed traffic, e.g. [1]-[3] and the references therein. Then the student will need to explore new control methods for regulating autonomous vehicles in mixed traffic. Another major focus will be the implementation of the analysis and control methods in Python. A range of numerical demonstration will be carried out in this project too.

[1] Zheng, Yang, Jiawei Wang, and Keqiang Li. "Smoothing traffic flow via control of autonomous vehicles." *IEEE Internet of Things Journal* 7.5 (2020): 3882-3896.

[2] Stern, Raphael E., et al. "Dissipation of stop-and-go waves via control of autonomous vehicles: Field experiments." *Transportation Research Part C: Emerging Technologies* 89 (2018): 205-221.

[3] Wang, J., Zheng, Y., Chen, C., Xu, Q., & Li, K. (2021). Leading cruise control in mixed traffic flow: System modeling, controllability, and string stability. *IEEE Transactions on Intelligent Transportation Systems*.

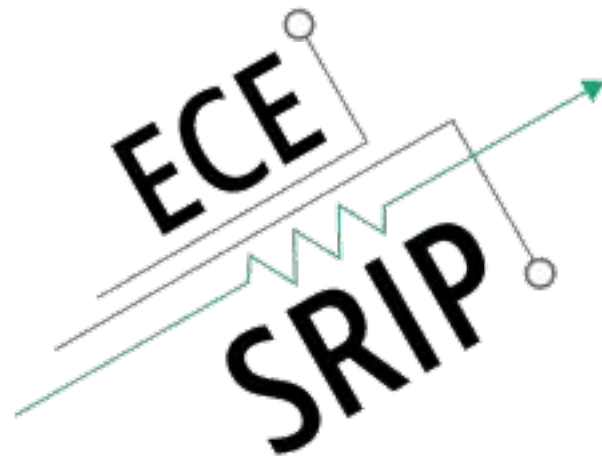
This project will be in person.

## **INTERNS NEEDED**

2 Students

## **PREREQUISITES**

1. A strong background in linear algebra and optimization, and good programming skills (Python, Matlab, Julia etc.) are required



## **FACULTY MENTOR**

Yang Zheng

## **PROJECT TITLE**

Optimization Landscape Analysis of Some Control Problems

## **PROJECT DESCRIPTION**

Description: Many recent studies have started to apply machine learning techniques to the control of unknown dynamical systems. They have achieved impressive empirical results. However, the convergence behavior, statistical properties, and robustness performance of these approaches are often poorly understood due to the nonconvex nature of the underlying control problems. In this project, we are interested in exploring the non-convex optimization landscape of some control problems, e.g., linear quadratic regular (LQR) [1], and linear quadratic Gaussian control (LQG) [2]. This will facilitate the understanding of model-free policy gradient algorithms for classical control problems.

[1] Fazel, M., Ge, R., Kakade, S., & Mesbahi, M. (2018, July). Global convergence of policy gradient methods for the linear quadratic regulator. In International Conference on Machine Learning (pp. 1467-1476). PMLR.

[2] Zheng, Y., Tang, Y., & Li, N. (2021). Analysis of the Optimization Landscape of Linear Quadratic Gaussian (LQG) Control. arXiv preprint arXiv:2102.04393.

This project will be in person.

## **INTERNS NEEDED**

1 Student

## **PREREQUISITES**

1. Good knowledge of linear algebra, control and optimization